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Discussion on 13.01.2027
Homework due on 20.01.2027 at 14:15

Exercise Sheet 10 in Scientific and Technical English WiSe 2026/27

The exercise sheets consist of in-class exercises and homework. The in-class exercises take place during the second half of the lecture time slots. The homework, which is optional and ungraded, can be submitted via the “Homework” section in Moodle. The homework is subject to peer review.

Unless indicated otherwise, generative artificial intelligence assistants such as ChatGPT may be used, as long as you acknowledge how you use them as specified by the Institute’s policy on plagiarism.¹ However, you may not use such tools to generate peer reviews for you. In addition, we strongly recommend that you do not use them to generate entire solutions, since that would defeat the purpose of the exercises.

In-class exercise 10-1 *Shortening an Abstract* The following 344-word abstract² is long and verbose.

We identify a tradeoff curve between the number of wheels on a train car, and the amount of track that must be installed in order to ensure that the train car is supported by the track at all times. The goal is to build an elevated track that covers some large distance ℓ , but that consists primarily of gaps, so that the total amount of feet of train track that is actually installed is only a small fraction of ℓ . In order so that the train track can support the train at all points, the requirement is that as the train drives across the track, at least one set of wheels from the rear quarter and at least one set of wheels from the front quarter of the train must be touching the track at all times.

We show that, if a train car has n sets of wheels evenly spaced apart in its rear and n sets of wheels evenly spaced apart in its front, then it is possible to build a train track that supports the train car but uses only $O(\frac{\ell}{n})$ feet of track. We then consider what happens if the wheels on the train car are not evenly spaced (and may even be configured adversarially). We show that for any configuration of the train car, with n wheels in each of the front and

¹<https://www.medien.ifi.lmu.de/lehre/Plagiate-IfI.pdf>

²William Kuszmaul, “Train Tracks with Gaps: Applying the Probabilistic Method to Trains,” *Theoretical Computer Science* 899, pp. 39–47, 2020.

rear quarters of the car, it is possible to build a track that supports the car for distance ℓ and uses only $O(\frac{\ell \log n}{n})$ feet of track. Additionally, we show that there exist configurations of the train car for which this tradeoff curve is asymptotically optimal. Both the upper and lower bounds are achieved via applications of the probabilistic method.

The algorithms and lower bounds in this paper provide simple illustrative examples of many of the core techniques in probabilistic combinatorics and randomized algorithms. These include the probabilistic method with alterations, the use of McDiarmid's inequality within the probabilistic method, the algorithmic Lovász Local Lemma, the min-hash technique, and the method of conditional probabilities.

- a) Shorten the abstract by at least 25% using the haircut and amputation approaches while preserving its essence, without using generative artificial intelligence assistants.
- b) Now use artificial intelligence to do the same for you. What is your prompt?
- c) Compare your solutions for tasks (a) and (b).
- d) Prepare a unified abstract that combines the best ideas from both versions.

Homework 10-2 *Incorporating Feedback* For your bachelor's thesis about a topic in computational geometry, you have written the following background section. It is designed to give a brief introduction to computational geometry problems and to serve as a motivating example for the algorithms you present in later sections.

Imagine you are walking on a university campus and suddenly you realize that you need to make an urgent phone call. There are many public phones on campus, and of course you want to go to the nearest one. But which one is the nearest? It would be helpful to have a map on which you could look up the nearest public phone, no matter where you are on campus. The map should show a subdivision of the campus into regions, and for each region indicate the nearest public phone. What would these regions look like? And how could we compute them?

This is just one example of a geometric problem requiring carefully designed geometric algorithms for their solution. The field of computational geometry emerged in the 70ies and deals with such geometric problems. It can be defined as the systematic study of algorithms and data structures for geometric objects.

To illustrate the issues that arise in developing a geometric algorithm, this section deals with one of the first problems that was studied in computational geometry: the computation of planar convex hulls.

A subset S of the plane is called convex if and only if for any pair of points $p, q \in S$, the line segment \overline{pq} is completely contained in S . The *convex hull* $\mathcal{CH}(S)$ of a set S is the smallest convex set that contains S . To be more precise, it is the intersection of all convex sets that contain S .

How do we compute the convex hull? Before we can answer this question, we must ask another question: What does it mean to compute the convex hull?

As we have seen, the convex hull of P is a convex polygon. A natural way to represent a polygon is by listing its vertices in clockwise order, starting with an arbitrary one. So the problem we want to solve is this: Given a set $P = \{p_1, p_2, \dots, p_n\}$ of points in the plane, compute a list that contains those points from P that are the vertices of $\mathcal{CH}(P)$, listed in clockwise order.

Algorithm 1 ConvexHull

Input:

A set P of points in the plane.

Output:

A list L containing the vertices of $\mathcal{CH}(P)$ in clockwise order.

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1:  $E \leftarrow \emptyset$ 
2: for all ordered pairs  $(p, q) \in P \times P$  with  $p$  not equal to  $q$  do
3:    $\text{valid} \leftarrow \text{true}$ 
4:   for all points  $r \in P$  not equal to  $p$  or  $q$  do
5:     if  $r$  lies to the left of the directed line from  $p$  to  $q$  then
6:        $\text{valid} \leftarrow \text{false}$ .
7:     end if
8:   end for
9:   if  $\text{valid}$  then
10:    add the directed edge  $\overrightarrow{pq}$  to  $E$ .
11:   end if
12: end for
13: From the set  $E$  of edges construct a list  $L$  of vertices of  $\mathcal{CH}(P)$ , sorted in clockwise order.
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Analyzing the time complexity of CONVEXHULL is easy. In the outer loop, we check $n^2 - n$ pairs of points. For each pair, we look at the $n - 2$ other points to see whether they all lie on the right side. This will take $O(n^3)$ time in total. An algorithm with a cubique running time is too slow to be of practical use for anything but the smallest input sets. To improve upon, this we apply a standard algorithmic design technique: an incremental algorithm.

Your supervisor and your cosupervisor have given you the following feedback in response to this section. Please edit the section in light of the feedback.

“The prose in the introduction is too flowery. All in all, you should try to write more formally.”

“Public phones are no longer relevant.”

“Algorithm 1 is a float, but it’s not referenced in the text.”

“I really like the beginning. It was a good informal motivation and fun to read.”

“There is a jarring break between the first part with the example and the second part with the algorithm. You should write some transition text.”

“There are some typos and spelling errors, like ‘covnex’. You should get into the habit of using a spellchecker.”