Cook's Theorem

Problem FSAT

Instance:	Formula F
Question:	Is F satisfiable ?

Theorem

FSAT *is NP-complete.*

The problem SAT

Problem k-SAT

Instance:	Formula F in k-CNF
Question:	Is F satisfiable ?

Theorem

For every formula F there is a formula $E(F)$ in 3-CNF s.t. $E(F)$ is satisfiable iff F is satisfiable.

Corollary

SAT and k-SAT for $k \geq 3$ are NP-complete.

Limiting occurrences

For a class $\mathcal{F}$ of formulas:
$\mathcal{F}(k)$: formulas in $\mathcal{F}$ with $\leq k$ occurrences of every variable.

Theorem

For every formula F in CNF there is a formula $D(F)$ in CNF(3) with $w(D(F)) = w(F)$ s.t. $D(F)$ is satisfiable iff F is satisfiable.

Corollary

3-SAT(3) is NP-complete.

More on limited occurrences

Ek-CNF: formulas with exactly k literals per clause

Proposition

E3-SAT(3) is trivial: all formulas in E3-CNF(3) are satisfiable.

OTOH: E3-SAT(4) is NP-complete.

More general: for every k, Ek-SAT(k) is trivial.

More on limited occurrences

Theorem

For every $k \geq 3$, there is $s = s(k) \geq k$ such that

- *Ek-SAT(s) is trivial*
- *Ek-SAT(s+1) is NP-complete.*

Bounds on the value $s(k)$:

k	$s(k) \geq$	$s(k) <$
3	3	4
4	4	5
5	5	8
6	7	12
7	13	18
8	24	30
9	41	52