Automated Theorem Proving

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Exercises 6: General Resolution

Exercise 6.1: Using the (a) standard and the (b) polynomial unification rules, compute most general unifiers of $P(g(x_1, x_1), g(x_3, h(x_4)))$ and $P(g(h(x_2), h(h(x_6))), g(h(x_5), x_5))$, if they exist.

Exercise 6.2: Using the (a) standard and the (b) polynomial unification rules, compute most general unifiers of $P(g(x_1, g(f(x_3), x_3)), g(h(x_4), x_3))$ and $P(g(x_2, x_2), g(x_3, h(x_1)))$, if they exist.

Exercise 6.3: In the lecture notes, standard unification ($\Rightarrow_{SU}$) is proved to be terminating based on a lexicographic combination of orderings. Can the same combination be used to prove the termination of polynomial unification ($\Rightarrow_{PU}$)?

Exercise 6.4: (a) Give an example of a most general unifier of $f(g(x, y))$ and $f(z)$ that is idempotent.

(b) Give an example of a most general unifier of $f(g(x, y))$ and $f(z)$ that is not idempotent.

Exercise 6.5: Let $\Sigma = (\Omega, \Pi)$ with $\Omega = \{b/0,\ c/0,\ f/2\}$ and $\Pi = \{P/1,\ Q/1,\ R/0\}$. Use the general resolution calculus *Res* to check whether the following clause set is satisfiable:

$$
\begin{aligned}
&\neg P(f(x,c)) \lor Q(x) && (1)\\
&\neg P(f(b,b)) \lor R && (2)\\
&\neg Q(b) \lor \neg R && (3)\\
&Q(c) \lor R && (4)\\
&P(f(b,y)) && (5)\\
&\neg P(c) && (6)
\end{aligned}
$$

Exercise 6.6: Let $\Sigma = (\Omega, \Pi)$ with $\Omega = \{b/0,\ f/1\}$ and $\Pi = \{P/1\}$. Use the general resolution calculus *Res* to determine whether the following clause set is satisfiable:

$$
\begin{aligned}
&P(x) \lor \neg P(f(x)) && (1)\\
&\neg P(b) && (2)
\end{aligned}
$$

Exercise 6.7 (∗)**:** For inferences with more than one premise, we implicitly assume that the variables in the premises are renamed such that they become different to any variable in the other premises. Show that the resolution calculus *without* this renaming is incomplete by exhibiting a saturated unsatisfiable clause set that does not contain the empty clause.

Exercise 6.8 (∗)**:** (a) Let N be a set of (not necessarily ground) first-order clauses. Let $D = \neg A$ be a negative unit clause such that no "Resolution" inference between any clause $C \in N$ and D is possible. Prove that no "Resolution" inference between any clause $C' \in \mathit{Res}^*(N)$ and D is possible.

(b) Does the property also hold if D is a positive unit clause or an arbitrary clause? Give a brief explanation.