Mockup Examination in the Course Automated Theorem Proving

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based on questions by Dr. Uwe Waldmann

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For convenience, a handout is provided with the definitions of the main proof calculi and concepts covered in the course.

Last name (in CAPITAL LETTERS):

First name (in CAPITAL LETTERS):

Matriculation number:

Program of study:

Hereby I confirm the correctness of the above information:

Signature

With your signature, you confirm that you are in sufficiently good health at the beginning of the examination and that you accept this examination bindingly.

Please leave the following table blank:

Question	1	2	3	4	5	6	$\sum$
Points	16	20	10	20	16	18	100
Score							

Instructions

You have **120 minutes** at your disposal. Written or electronic aids are not permitted except for normal watches. Carrying forbidden devices, even if they are turned off, will be considered a cheating attempt.

Write your full name and matriculation number clearly legible on this cover sheet, as well as your name in the header on each sheet. Hand in all sheets. Leave them stapled together. Use only **pens** and **neither** the color **red nor green**.

There are 6 questions for a total of 100 points. The questions can be found on **pages 3–8**. The questions can be completed independently of each other. Check that you have received all the sheets and the handout.

You may use the back of the sheets for secondary calculations. If you use the back of a sheet to answer, clearly mark what belongs to which question, and indicate in the corresponding question where all parts of your answer can be found. Cross out everything that should not be graded.

Provide a single answer to each question and subquestion. If multiple answers are provided, the worst one will be graded.

Write legibly. If an answer is illegible, it will be interpreted disfavorably for grading.

Question 1 (16 points)**:** Let F be the first-order formula

$$\exists z \, \forall x \, ((\exists y \, P(z, y) \rightarrow \exists y \, Q(x, y)))$$

(a) [12 points] Transform F into clausal normal form. Proceed one step at a time. Indicate each step.

(b) [4 points] Are the formula F and its clausal normal form equivalent? Explain your answer.

Question 2 (20 points)**:** Let N be the following set of propositional clauses over $\Pi = \{P, Q, R\}$:

$$P \lor \neg Q \qquad (1)$$
$$P \lor \neg R \qquad (2)$$
$$\neg P \lor R \qquad (3)$$

(a) [10 points] Use the DPLL procedure to find a (total) model of N. Document each step of the procedure.

(b) [10 points] Use the DPLL procedure to prove that $N \models Q \rightarrow R$. You will first need to transform the entailment into a formula to refute. Document each step of the procedure.

Question 3 (10 points)**:** Let $\Sigma = (\Omega, \emptyset)$ with $\Omega = \{f/1,\ g/1,\ h/2,\ b/0,\ c/0\}$. Find a total precedence $\succ$ on Ω such that the lexicographic path ordering $\succ_{\text{lpo}}$ that is induced by $\succ$ satisfies the following three properties simultaneously.

$$h(x, f(y)) \succ_{\text{lpo}} h(g(y), y) \qquad (1)$$

$$h(x, c) \succ_{\text{lpo}} g(h(x, b)) \qquad (2)$$

$$g(b) \succ_{\text{lpo}} c \qquad (3)$$

There is no need to explain your answer.

Question 4 (20 points)**:** Let $\Sigma = (\{f/1, g/2, b/0, c/0\}, \{P/2, Q/1, R/2\})$. Let the atom ordering $\succ$ be the kbo with weight 1 for all symbols and variables and the precedence $P \succ Q \succ R \succ f \succ g \succ b \succ c$. Let N be the following set of clauses over Σ:

$$\neg R(b, x) \qquad (1)$$

$$Q(g(x, z)) \lor Q(g(b, f(y))) \lor \neg R(x, z) \qquad (2)$$

$$P(f(x), x) \lor \neg Q(f(c)) \lor \neg R(x, c) \qquad (3)$$

$$\neg P(x, c) \lor Q(x) \qquad (4)$$

$$\neg P(x, b) \lor R(c, y) \qquad (5)$$

(a) [16 points] Suppose that the selection function *sel* selects no literals. Compute all $\text{Res}_{sel}^{\succ}$ inferences between the clauses (1)–(5). Do not compute inferences between derived clauses.

(b) [4 points] Is the set N saturated up to redundancy? Explain your answer.

Question 5 (16 points)**:** Let R be the following set of rewrite rules over $\Sigma = (\{f/1, g/2, h/1, c/0\}, \emptyset)$:

$$\begin{aligned} f(f(x)) &\to h(h(x)) && (1) \\ g(f(y), x) &\to g(y, x) && (2) \\ h(g(z, f(c))) &\to f(z) && (3) \end{aligned}$$

There are exactly three critical pairs between the three rules. Find them.

Question 6 (18 points)**:** Let $\Sigma = (\{f/1,\ g/1,\ b/0\}, \emptyset)$. Consider the set N consisting of the following of equational clauses over Σ:

$$g(b) \approx b$$
$$g(x) \not\approx b \vee g(f(x)) \approx b$$

We use the superposition calculus without selection and the kbo with $g \succ f \succ b$ and weight 1 for symbols and variables as the term ordering.

(a) [4 points] Is the set N saturated up to redundancy? If yes, briefly explain why. If no, give an inference from N that is not redundant w.r.t. N.

(b) [14 points] Complete the following table documenting the first four iterations of the candidate interpretation construction leading to the rewrite system R_∞ for the set of ground clauses $G_\Sigma(N)$.

Iter.	Clause C	R_C	E_C
0	$g(b) \approx b$	$\emptyset$	
1	$g(b) \not\approx b \vee g(f(b)) \approx b$		
2	$g(f(b)) \not\approx b \vee g(f(f(b))) \approx b$		
3	$g(g(b)) \not\approx b \vee g(f(g(b))) \approx b$		