Automated Theorem Proving

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based on exercises by Dr. Uwe Waldmann

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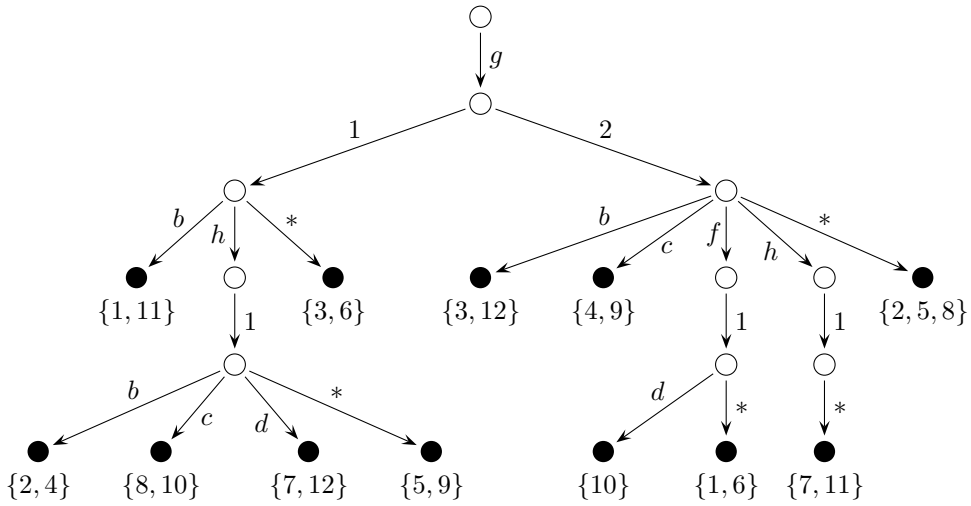
Exercises 14: Efficient Saturation Procedures and Outlook

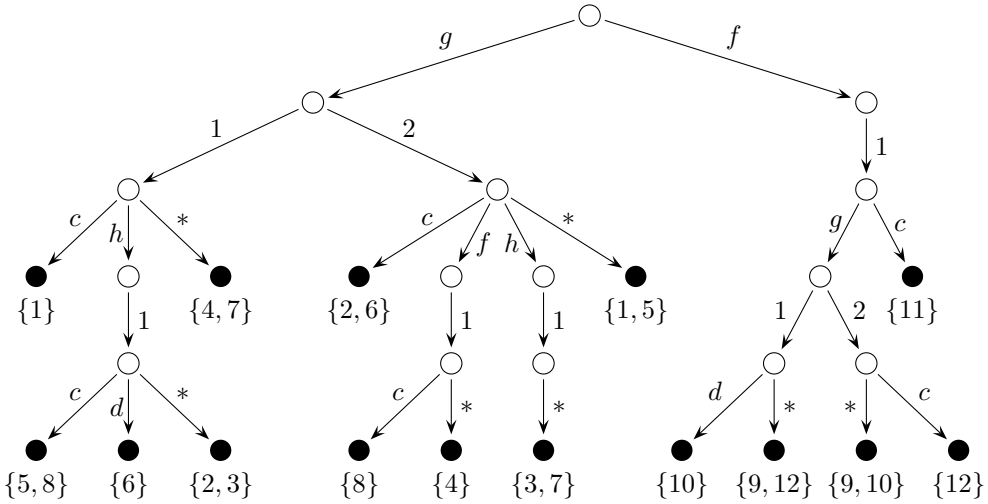
Exercise 14.1: Consider the following path index:



Does path index contain the terms $t_1 = g(h(d), h(*))$, $t_2 = g(h(b), c)$, $t_3 = g(*, *)$? If yes, what are their numbers in the index?

Proposed solution. The path index contains t_1 with index 7 and t_2 with index 4. It does not contain t_3.

Exercise 14.2: Consider the following path index:



(a) Which terms have the numbers 3, 5, and 12 in the path index?

(b) Which of the terms $g(*, h(*))$, $f(g(d, c))$, and $g(h(*), c)$ are contained in the path index? If they are contained, what are their numbers?

(c) Assume that the terms in the path index are the left-hand sides of the rewrite rules of a TRS R. Is the term $f(g(h(d), f(c)))$ reducible by rules in R? If yes, what are the numbers of the left-hand sides of these rules?

Proposed solution. (a) term 3: $g(h(*), h(*))$; term 5: $g(h(c), *)$; term 12: $f(g(*, c))$.

(b) $g(*, h(*))$: term 7; $f(g(d, c))$: not contained in the index; $g(h(*), c)$: term 2.

(c) $f(g(h(d), f(c)))$ is reducible by the rules whose left-hand sides have the numbers 9, 4, and 11.

Exercise 14.3: Could one use the following numbers as features in a feature vector index?

(1) the number of ground arguments of predicate symbols in a clause,

(2) the number of variable occurrences in a clause,

(3) the number of constant symbols occurring in positive literals in a clause,

(4) the number of literals in a clause that do not contain variables,

(5) the number of literals in a clause that do not contain the function symbol f,

(6) the number of literals in a clause that do not contain the predicate symbol P,

(7) the number of literals in a clause that contain neither variables nor the function symbol f,

(8) the number of distinct variables in a clause.

Proposed solution. (1), (3), (4), (5), (6), (7) can be used.

(2) cannot be used. For example, $P(f(b))$ is subsumed by $P(x)$ and contains fewer variable occurrences than the subsumer, but $P(g(y,z)))$ is subsumed by $P(x)$ and contains more variable occurrences than the subsumer.

(5) cannot be used. $P(f(x))$ is subsumed by $P(x)$ despite the latter having more (one) literals that contain the function symbol f than the former (which has zero such literals). Conversly we cannot use this critera in reverse because the clause $P(x)$ subsumes the clause $P(x) \vee Q(x)$ despite the latter having more literals that do not contain the function symbol f than the former.

(8) cannot be used. The same counterexample as for (2) applies here as well.