Retake Examination in the Course Automated Theorem Proving

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For convenience, a handout is provided with the definitions of the main calculi and concepts covered in the course.

Last name (in CAPITAL LETTERS):

First name (in CAPITAL LETTERS):

Matriculation number:

Program of study:

Hereby, I confirm the correctness of the above information:

Signature

With your signature, you confirm that you are sufficiently healthy at the beginning of the examination and that you accept the examination bindingly.

Please leave the following table blank:

Question	1	2	3	4	5	6	$\sum$
Points	20	17	12	20	15	16	100
Score							

Instructions

You have **120 minutes** at your disposal. Written or electronic aids are not permitted except for normal watches. Carrying forbidden devices, even turned off, will be considered a cheating attempt.

Write your full name and matriculation number clearly legible on this cover sheet, as well as your name in the header on each sheet. Hand in all sheets. Leave them stapled together. Use only **pens** and **neither** the color **red nor green**.

Check that you have received all the sheets and the handout. The questions can be found on **pages 3–10**. You may use the back of the sheets for secondary calculations. If you use the back of a sheet to answer, clearly mark what belongs to which question, and indicate in the corresponding question where all parts of your answer can be found. Cross out everything that should not be graded.

There are 6 questions for a total of 100 points.

Question 1 (20 points)**:** Consider the first-order formula $F = \forall x, y\, ((P(x) \land P(y)) \rightarrow P(x))$ over $\Sigma = (\{b/0\}, \{P/1\})$.

(a) Transform the negation of F into clausal normal form. Proceed one step at a time, and record each step.

(b) Recall that $F = \forall x, y\,((P(x) \wedge P(y)) \rightarrow P(x))$ and $\Sigma = (\{b/0\}, \{P/1\})$. Use the tableau calculus with ground instantiation to prove F by refutation. Use exactly the expansion rules seen in the lecture; do not preclausify the problem. Document all your steps.

Question 2 (17 points)**:** Let $\Sigma = (\Omega, \emptyset)$ with $\Omega = \{zero/0,\ succ/1,\ add/2,\ mul/2\}$, let x, y be variables, and let R be the following rewrite system:

$$\begin{aligned} mul(zero, y) &\to zero && (1)\\ mul(succ(x), y) &\to add(y, mul(x, y)) && (2)\\ add(zero, y) &\to y && (3)\\ add(succ(x), y) &\to succ(add(x, y)) && (4) \end{aligned}$$

(a) Specify the precedence of an lpo $\succ$ such that the rewrite rules (1)–(4) are oriented from left to right. There is no need to explain your answer.

(b) Specify the precedence and the weights of a kbo $\succ$ such that the rewrite rules (1), (3), and (4) are oriented from left to right. There is no need to explain your answer.

(c) Compute all critical pairs between rules in R and determine whether they are joinable in R.

Question 3 (12 points)**:** Refute the following set of equational clauses using the superposition calculus:

$$e \not\approx zero \quad (1)$$

$$x \approx zero \lor div(one, x) \approx inv(x) \quad (2)$$

$$abs(div(one, e)) \not\approx abs(inv(e)) \quad (3)$$

Use the kbo with the precedence $abs \succ div \succ inv \succ one \succ e \succ zero$, weight 100 for $zero$, weight 10 for e, and weight 1 for all other symbols and variables. Document all of your steps.

Question 4 (20 points)**:** Let N be the following set of propositional clauses over $\Pi = \{P, Q, R\}$:

$$\neg P \lor \neg Q \quad (1)$$

$$\neg P \lor \neg Q \lor \neg R \quad (2)$$

$$P \lor R \quad (3)$$

(a) Use the DPLL procedure to find a model of N. Document each step of the procedure.

(b) Use the DPLL procedure to prove that $N \models \neg Q \lor R$. You will first need to transform the entailment into a formula to refute. Document each step of the procedure.

Question 5 (15 points)**:** For this question, we use the signature $\Sigma = (\{f/1,\ g/1,\ b/0\}, \{P/1\})$, the clause set $N = \{P(g(x))\}$ over Σ, and the kbo with $P \succ g \succ f \succ b$ and weights 1 for all symbols and variables.

(a) Complete the following table summarizing the first five iterations of the candidate interpretation construction. Hint: Recall that the clauses considered in the second column must be groundings of clauses in N.

Iter.	Clause C	I_C	Δ_C
0	$P(g(b))$	$\emptyset$	$\{P(g(b))\}$
1			
2			
3	$P(g(f(f(b))))$		
4			

(b) Determine I_∞ for the clause set N.

Question 6 (16 points)**:** We call a ground clause *symmetric* if for every literal L it contains, it also contains the complementary literal $\overline{L}$. Thus, $P(b) \vee Q(c) \vee \neg P(b) \vee \neg Q(c)$ is symmetric.

(a) Prove that every inference of the ground resolution calculus from symmetric premises generates a symmetric conclusion.

(b) Exhibit a nonempty set of symmetric clauses that is satisfiable. Briefly explain why the set is satisfiable.

(c) Exhibit a set of symmetric clauses that is unsatisfiable. Briefly explain why the set is unsatisfiable.