Regular Examination in the Course Automated Theorem Proving

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For convenience, a handout is provided with the definitions of the main calculi and concepts covered in the course.

Last name (in CAPITAL LETTERS):

First name (in CAPITAL LETTERS):

Matriculation number:

Program of study:

Hereby, I confirm the correctness of the above information:

Signature

With your signature, you confirm that you are sufficiently healthy at the beginning of the examination and that you accept the examination bindingly.

Please leave the following table blank:

Question	1	2	3	4	5	6	$\sum$
Points	17	21	16	16	12	18	100
Score							

Instructions

You have **120 minutes** at your disposal. Written or electronic aids are not permitted except for normal watches. Carrying forbidden devices, even turned off, will be considered a cheating attempt.

Write your full name and matriculation number clearly legible on this cover sheet, as well as your name in the header on each sheet. Hand in all sheets. Leave them stapled together. Use only **pens** and **neither** the color **red nor green**.

Check that you have received all the sheets and the handout. The questions can be found on **pages 3–13**. You may use the back of the sheets for secondary calculations. If you use the back of a sheet to answer, clearly mark what belongs to which question, and indicate in the corresponding question where all parts of your answer can be found. Cross out everything that should not be graded.

There are 6 questions for a total of 100 points.

Question 1 (17 points)**:** (a) We call a clause *depressed* if it contains at least as many negative literals as it contains positive literals. For example, $P(a) \vee \neg Q(b) \vee \neg R$ and $P(a) \vee \neg Q(b) \vee \neg R \vee S$ are depressed. Prove or disprove that every inference of the resolution calculus from depressed premises generates a depressed conclusion.

(b) We call a clause *balanced* if it contains exactly as many negative literals as it contains positive literals. Prove or disprove that every inference of the resolution calculus from balanced premises generates a balanced conclusion.

(c) We call a clause *unbalanced* if it is not balanced (as per subquestion (b)). Prove or disprove that every inference of the resolution calculus from unbalanced premises generates an unbalanced conclusion.

Question 2 (21 points)**:** Consider the following entailment between two propositional formulas:

$$P \rightarrow (Q \vee R) \models (P \wedge \neg Q) \rightarrow R$$

(a) Express the entailment as a set of clauses by negating the conjecture and using the CNF transformation. Document each step.

(b) Let $N = \{\neg S \vee T \vee U, S, \neg T, \neg U\}$ be a clause set. Use the DPLL procedure to show whether N is satisfiable or unsatisfiable. Document each DPLL step.

(c) Use the (unordered) resolution calculus to derive the empty clause from the clause set $N = \{\neg S \vee T \vee U,\ S,\ \neg T,\ \neg U\}$. Document all of your steps.

(d) Use the ground tableau calculus to prove the entailment

$$P \to (Q \lor R) \models (P \land \neg Q) \to R$$

You will first need to transform the entailment into a formula to refute. Use exactly the expansion rules seen in the lecture; do not preclausify the problem. Document all of your steps.

Question 3 (16 points)**:** Let $\Sigma = (\{append/2,\ cons/2,\ nil/0,\ rev/1\},\ \emptyset)$, and let x, xs, ys be variables. Consider the following set of equations:

$$\begin{aligned} rev(nil) &\approx nil \\ rev(cons(x, xs)) &\approx append(rev(xs), cons(x, nil)) \\ append(nil, ys) &\approx ys \\ append(cons(x, xs), ys) &\approx cons(x, append(xs, ys)) \end{aligned}$$

(a) Specify a precedence for the lpo so that the left-hand side of each equation in the set is greater than the corresponding right-hand side according to that instance of the lpo. There is no need explain your answer.

(b) Note that the kbo cannot be used to orient these equations from left to right. Regardless of the weights chosen, the right-hand side of the second equation has a greater weight than the left-hand side. So let us ignore the second equation.

Specify a precedence, symbol weights, and a variable weight for the kbo so that the left-hand side of each of the *first, third, and fourth* equations is greater than the corresponding right-hand side according to that instance of the kbo. There is no need explain your answer.

Question 4 (16 points)**:** Recall that a relation $\rightarrow$ is called

terminating if there is no infinite descending chain $b_0 \rightarrow b_1 \rightarrow b_2 \rightarrow \cdots$;

normalizing if every $b \in A$ has a normal form;

locally confluent if $b \leftarrow a \rightarrow c$ implies there is a d such that $b \rightarrow^* d \leftarrow^* c$;

confluent if $b \leftarrow^* a \rightarrow^* c$ implies there is a d such that $b \rightarrow^* d \leftarrow^* c$.

Let

$$\{f(x, y) \rightarrow f(y, x),\ f(x, y) \rightarrow d\}$$

be a rewrite system over $\Sigma = (\{f/2,\ b/0,\ c/0,\ d/0\},\ \emptyset)$. Is the system (a) terminating? (b) normalizing? (c) locally confluent? (d) confluent? Briefly explain each of your answers.

Question 5 (12 points)**:** Refute the following set of equational clauses using the superposition calculus:

$$pi \not\approx zero \quad (1)$$

$$x \approx zero \vee div(one, x) \approx inv(x) \quad (2)$$

$$abs(div(one, pi)) \not\approx abs(inv(pi)) \quad (3)$$

Use the kbo with the precedence $abs \succ div \succ inv \succ one \succ pi \succ zero$ and weight 1 for all symbols and variables. Document all of your steps.

Question 6 (18 points)**:** For this question, we use the signature $\Sigma = (\{f/1,\ g/1,\ b/0\}, \emptyset)$, the clause set $N = \{g(x) \approx b\}$ over Σ, and the kbo with $g \succ f \succ b$ and weights 1 for all symbols and variables.

(a) Complete the following table summarizing the first six iterations of the candidate interpretation construction. Hint: Recall that the clauses considered in the second column must be groundings of clauses in N.

Iter.	Clause C	R_C	E_C
0	$g(b) \approx b$	$\emptyset$	$\{g(b) \to b\}$
1			
2			
3	$g(f(f(b))) \approx b$		
4			
5			

(b) Give a model of N.