Automated Theorem Proving

Prof. Dr. Jasmin Blanchette, Yiming Xu, PhD,
Tanguy Bozec, and Lydia Kondylidou
based on exercises by Dr. Uwe Waldmann

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Exercises 11: Completion

Exercise 11.1: Apply the Knuth–Bendix procedure to the set of equations

$$f(f(x)) \approx g(x) \qquad (1)$$
$$f(b) \approx c \qquad (2)$$

and transform it into a finite convergent term rewrite system. Use the Knuth–Bendix ordering with weight 1 for all function symbols and variables and the precedence $g \succ f \succ b \succ c$.

Exercise 11.2: Let E be the following set of equations over $\Sigma = (\{f/1,\ g/1,\ h/1,\ b/0\}, \emptyset)$.

$$f(g(x)) \approx h(x) \qquad (1)$$
$$g(f(x)) \approx h(x) \qquad (2)$$

Apply the Knuth–Bendix completion procedure to E and transform it into a finite convergent term rewrite system. Use a Knuth–Bendix ordering with weight 1 for all function symbols and variables and the precedence $f \succ g \succ h \succ b$. Use a reasonable strategy.

Exercise 11.3: Let E be the following set of equations over $\Sigma = (\{f/2,\ g/1,\ b/0,\ c/0\}, \emptyset)$.

$$f(x,x) \approx f(x,b) \qquad (1)$$
$$f(x,x) \approx f(c,x) \qquad (2)$$
$$f(x,x) \approx g(x) \qquad (3)$$

Apply the Knuth–Bendix completion procedure to E and transform it into a finite convergent term rewrite system. Use a Knuth–Bendix ordering with weight 1 for all function symbols and variables and the precedence $f \succ g \succ b \succ c$. Use a reasonable strategy.

Exercise 11.4: Let E be the following set of equations over $\Sigma = (\{f/1,\ g/1,\ h/1\}, \emptyset)$.

$$f(g(f(x))) \approx h(x) \qquad (1)$$
$$g(h(x)) \approx x \qquad (2)$$

Apply the Knuth–Bendix completion procedure to E and transform it into a finite convergent term rewrite system. Use the Knuth–Bendix ordering with weight 1 for all function symbols and variables and the precedence $f \succ g \succ h$. Use a reasonable strategy.

Exercise 11.5: Let E be the following set of equations over $\Sigma = (\{f/2,\ g/1,\ h/1,\ b/0\}, \emptyset)$.

$$f(g(x), x) \approx b \qquad (1)$$
$$f(x, b) \approx x \qquad (2)$$
$$g(h(x)) \approx x \qquad (3)$$

Apply the Knuth–Bendix completion procedure to E and transform it into a finite convergent term rewrite system. Use a Knuth–Bendix ordering with weight 1 for all function symbols and variables and the precedence $f \succ g \succ h \succ b$. Use a reasonable strategy.