A major problem in the area of propositional proof complexity is the lack of candidate formulas that could be shown to require long proofs in strong proof systems, in particular in textbook-style axiomatic systems (called *Frege systems* in this context) or, equivalently, the propositional fragment of Gentzen's LK.

Stephen A. Cook proposed certain matrix identities, the so-called "hard matrix identities", most notably the implication $AB = I \rightarrow BA = I$, as candidates for such hard formulas. In order to study the proof complexity of these matrix identities, the author and S. A. Cook [1] introduced a logical system **LA** for reasoning about matrices, and several extesions of it.

The paper under review continues this line of work. It is shown that in **LA** over finite fields, from (a matrix form of) the pigeonhole principle one can derive the hard matrix identities.

The theory **LAP** is **LA** extended with a function symbol for matrix powering and its defining axioms. An extension of **LAP** by quantification over permutation matrices, called $\exists P$**LAP**, is shown to be strong enough to prove fundamental theorems of linear algebra such as the Cayley-Hamilton Theorem. This implies that in particular $\exists P$**LAP** proves the hard matrix identities.

Furthermore, it is shown that $\exists P$**LA**, that is **LA** with quantification over permutation matrices (but without the powering operation), can express graph-theoretic properties in the complexity class NP, such as the property of having a Hamiltonian Path, and k-Colorability. The theory $\exists P$**LA** also proves the soundness of the Hajós Calculus, a proof system for generating non-3-colorable graphs.

The paper concludes by stating several open problems.

References

[1] M. Soltys and S. Cook, The proof complexity of linear algebra, Ann. Pure Appl. Logic **130** (2004), no. 1-3, 277–323. MR2092854 (2005g:03099)