The hierarchies of theories S_2^i and T_2^i of Bounded Arithmetic were defined by Buss [1], for $i \geq 1$ they are closely related to computational complexity classes in the polynomial time hierarchy.

The theories S_2^0 and T_2^0 allow induction on binary notation, resp. successor induction only for sharply bounded formulas, i.e., formulas in which every quantifier is bounded by a logarithmic term. The theory S_2^0 was shown to be pathologically weak by Takeuti [3], and it was generally believed that T_2^0 was likewise weak.

The paper under review studies the theory T_2^0 in the language of Bounded Arithmetic extended by a function symbol for $MSP(x,i) = \lfloor x/2^i \rfloor$. This function gives access to the binary representation of numbers, so the extended language is very natural and frequently used in the context of Bounded Arithmetic. The reviewer [2] has shown that S_2^0 in this extended language is still weak.

In this paper it is shown that T_2^0 in this language is surprisingly strong: it allows to define all polynomial time computable functions, and proves induction for sharply bounded formulas containing function symbols for these. Hence PV_1, the universal theory of polynomial time computable functions, is a conservative extension of T_2^0.

References

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