In this paper, a new propositional proof system $H$ is introduced, that allows quantification over permutations of the variables. In $H$ the syntax of propositional logic is enriched by quantifiers $(\exists ab)\alpha$ and $(\forall ab)\alpha$ for variables $a$ and $b$, which are intended to be semantically equivalent to $\alpha \lor \alpha[b/a, a/b]$ and $\alpha \land \alpha[b/a, a/b]$, respectively.

The paper studies the fragment of $H$ with cuts restricted to $\Sigma_1$-formulas, denoted $H_1$. It is shown that $H_1$ simulates efficiently the Hajós calculus ($HC$) for constructing graphs which are non-3-colorable. This shows that short proofs using formulas asserting the existence of permutations of the variables can capture polynomial time reasoning, as it is known [1] that $HC$ is equivalent to Extended Frege systems ($EF$), which capture polynomial time reasoning.

The converse direction is left open, but it is shown that at least $EF$ efficiently simulates tree-like proofs in $H_1$.

References