This paper develops model-theoretic techniques to obtain conservation results for first order Bounded Arithmetic theories, based on a hierarchical version of the model-theoretic notion of an existentially closed model.

For every theory axiomatized by an axiom scheme – like the induction scheme – there are weaker variants where the scheme is only given as an inference rule, and where it is only applied to parameter-free formulas.

Buss' theories T_2^i and S_2^i are defined by the induction scheme, resp. the scheme of induction on notation for Σ_i^b-formulas, they are strongly related to the polynomial time hierarchy. It is shown that these theories are $\forall\Sigma_i^b$-conservative over their versions given by an inference rule, and $\exists\forall\Sigma_i^b$-conservative over their parameter-free versions. Similar results are also shown for the theories axiomatized by the Σ_i^b-replacement scheme.

The proof method is essentially the same for all the schemes considered and shows that these conservation results between schemes and inference rules do not depend on the specific combinatorial or arithmetical content of those schemes. Hence, such conservation results can be derived very generally, for every axiom scheme enjoying certain logical conditions defined in this paper, which are common to both the induction and replacement schemes. Hence, the above conservation results for the induction and replacement schemes can be obtained as corollaries of these more general results.

Finally, the paper contains some additional conservation results concerning weak theories of Bounded Arithmetic that are related to the small computational complexity class TC^0.