It is well-known that theories of Bounded Arithmetic are closely related to propositional proof systems. This relation can be utilized in both directions: upper bounds and simulations for propositional proof systems can be shown by constructing proofs in the corresponding theories, and independence results for certain theories can be proven via lower bounds on the length of propositional proofs.

This survey paper explains and develops the general correspondence between propositional proof systems and arithmetic theories, as introduced by Krajíček and Pudlák [1]. Instead of focussing on particular pairs of proof systems and theories, a general axiomatic approach to the correspondence is presented. A particularly emphasis is put on the role played by logical closure properties of propositional proof systems. This also yields a new characterization of extensions of Extended Frege systems in terms of simple combinations of these closure properties.

References

[1] J. Krajíček and P. Pudlák, Quantified propositional calculi and fragments of bounded arithmetic, Z. Math. Logik Grundlag. Math. **36** (1990), no. 1, 29–46. MR1030537 (91f:03118)