Automated Theorem Proving

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based on exercises by Dr. Uwe Waldmann

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Exercises 12: Superposition

Exercise 12.1: Refute the following set of equational clauses by superposition:

$$x \approx b \lor x \approx c \lor x \approx d \quad (1)$$
$$e \not\approx b \quad (2)$$
$$e \not\approx c \quad (3)$$
$$e \not\approx d \quad (4)$$

Choose an appropriate ordering and perform only inferences that satisfy the ordering restrictions.

Exercise 12.2: Refute the following set of equational clauses by superposition:

$$f(x) \not\approx c \lor f(x) \approx b \quad (1)$$
$$f(f(x)) \approx x \quad (2)$$
$$b \not\approx c \quad (3)$$

Choose an appropriate ordering and perform only inferences that satisfy the ordering restrictions.

Exercise 12.3: Consider the following set of equational clauses:

$$f(b) \approx \mathit{true} \quad (1)$$
$$f(x) \not\approx \mathit{true} \lor f(g(x)) \approx \mathit{true} \quad (2)$$

(a) Saturate this set by computing superposition inferences *ignoring ordering restrictions*.

(b) Choose an appropriate ordering and perform only inferences that satisfy the ordering restrictions.

Exercise 12.4: Prove that the ground "Equality Resolution" inference rule is sound:

Equality Resolution: $$\frac{C' \vee s \not\approx s}{C'}$$

Exercise 12.5: Prove that the ground "Equality Factoring" inference rule is sound:

Equality Factoring: $$\frac{C' \vee s \approx t' \vee s \approx t}{C' \vee t \not\approx t' \vee s \approx t'}$$

Exercise 12.6: Prove that the ground "Negative Superposition" inference rule is sound:

Neg. Superposition: $$\frac{D' \vee t \approx t' \qquad C' \vee s[t] \not\approx s'}{D' \vee C' \vee s[t'] \not\approx s'}$$

Exercise 12.7: In the lecture notes, it is stated that the ordering restrictions of the inference rules of the superposition calculus must be satisfied *after applying the mgu to the premises*. Give a simple example that shows that a literal may be maximal in a clause but that the maximality requirement may be violated after applying the mgu.

Exercise 12.8 $(*)$**:** Find a small unsatisfiable set N of equational clauses and a term ordering $\succ$ such that N is saturated w.r.t. the superposition calculus *excluding* the "Equality Factoring" rule and N does not contain $\bot$. The existence of such a set implies that the superposition calculus is incomplete without "Equality Factoring."

Hint: Recall the informal motivation for adding "Equality Factoring" to the calculus.