Automated Theorem Proving

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based on exercises by Dr. Uwe Waldmann

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Exercises 11: Completion

Exercise 11.1: Apply the Knuth–Bendix procedure to the set of equations

$$f(f(x)) \approx g(x) \qquad (1)$$
$$f(b) \approx c \qquad (2)$$

and transform it into a finite convergent term rewrite system. Use the Knuth–Bendix ordering with weight 1 for all function symbols and variables and the precedence $g \succ f \succ b \succ c$.

Exercise 11.2: Let E be the following set of equations over $\Sigma = (\{f/1,\ g/1,\ h/1,\ b/0\}, \emptyset)$.

$$f(g(x)) \approx h(x) \qquad (1)$$
$$g(f(x)) \approx h(x) \qquad (2)$$

Apply the Knuth–Bendix completion procedure to E and transform it into a finite convergent term rewrite system. Use a Knuth–Bendix ordering with weight 1 for all function symbols and variables and the precedence $f \succ g \succ h \succ b$. Use a reasonable strategy.

Exercise 11.3: Let E be the following set of equations over $\Sigma = (\{f/2,\ g/1,\ b/0,\ c/0\}, \emptyset)$.

$$f(x, x) \approx f(x, b) \qquad (1)$$
$$f(x, x) \approx f(c, x) \qquad (2)$$
$$f(x, x) \approx g(x) \qquad (3)$$

Apply the Knuth–Bendix completion procedure to E and transform it into a finite convergent term rewrite system. Use a Knuth–Bendix ordering with weight 1 for all function symbols and variables and the precedence $f \succ g \succ b \succ c$. Use a reasonable strategy.

Exercise 11.4: Let E be the following set of equations over $\Sigma = (\{f/1,\ g/1,\ h/1\}, \emptyset)$.

$$f(g(f(x))) \approx h(x) \qquad (1)$$
$$g(h(x)) \approx x \qquad (2)$$

Apply the Knuth–Bendix completion procedure to E and transform it into a finite convergent term rewrite system. Use the Knuth–Bendix ordering with weight 1 for all function symbols and variables and the precedence $f \succ g \succ h$. Use a reasonable strategy.

Exercise 11.5: Let E be the following set of equations over $\Sigma = (\{f/2,\ g/1,\ h/1,\ b/0\}, \emptyset)$.

$$f(g(x), x) \approx b \qquad (1)$$
$$f(x, b) \approx x \qquad (2)$$
$$g(h(x)) \approx x \qquad (3)$$

Apply the Knuth–Bendix completion procedure to E and transform it into a finite convergent term rewrite system. Use a Knuth–Bendix ordering with weight 1 for all function symbols and variables and the precedence $f \succ g \succ h \succ b$. Use a reasonable strategy.

Exercise 11.6: Let $\Sigma = (\Omega, \emptyset)$ with $\Omega = \{f/4,\ b/0,\ c/0,\ d/0,\ e/0\}$. Let $\succ$ be the lpo with the precedence $f \succ b \succ c \succ d \succ e$. Let E be the set of equations

$$f(w, x, y, z) \approx f(x, y, z, w) \qquad (1)$$
$$f(c, d, e, b) \approx b \qquad (2)$$
$$f(c, b, e, d) \approx c \qquad (3)$$

Compute the set of semicritical pairs $\mathrm{SC}_{\succ}(E)$.

Exercise 11.7: Use unfailing completion to transform the set of equations

$$b \approx c \qquad (1)$$
$$b + d \approx e \qquad (2)$$
$$x + y \approx y + x \qquad (3)$$

into a ground convergent set of equations. Use the lpo with the precedence $+ \succ b \succ c \succ d \succ e$.