Automated Theorem Proving

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based on exercises by Dr. Uwe Waldmann

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Exercises 13: Superposition Continued

Exercise 13.1: Using the kbo with $f \succ b \succ c \succ d \succ e$ and weight 1 for all symbols and variables as the term ordering, compute the rewrite systems R_C and R_∞ for the set of ground clauses N:

$$f(b) \approx e \vee f(b) \not\approx f(b) \qquad (1)$$
$$b \not\approx e \vee f(c) \approx f(e) \qquad (2)$$
$$f(d) \approx f(e) \qquad (3)$$
$$f(e) \approx e \vee f(e) \approx c \qquad (4)$$
$$b \approx c \qquad (5)$$
$$d \approx e \qquad (6)$$

Which is the smallest clause $C \in N$ such that C is neither productive nor true in R_C? Use it to show that N is not saturated up to redundancy.

Exercise 13.2: Compute R_∞ for the clause set $\{f(x) \approx b\}$ and the signature $\Sigma = (\{f/1,\ g/1,\ b/0\}, \emptyset)$. Use the kbo with $g \succ f \succ b$ and weights 1 for all symbols and variables.

Exercise 13.3: Compute R_∞ for the clause set $\{f(x) \approx b\}$ and the signature $\Sigma = (\{f/1,\ g/1,\ b/0\}, \emptyset)$. This time, use the lpo with the precedence $g \succ f \succ b$.

Exercise 13.4: Let N be a set of equational clauses such that $\bot \notin N$. In Thm. 5.4.8, we have shown that whenever N is saturated up to redundancy, then every ground instance $C\theta \in G_\Sigma(N)$ is either productive or true in $R_{C\theta}$. The converse does not hold, not even for ground unit clauses: Give a small set of ground unit clauses N such that $\bot \notin N$ and every $C \in N$ is either productive or true in R_C, but N is not saturated up to redundancy.

Exercise 13.5: A clause is called *Horn* if it contains at most one positive literal. Prove that every inference of the superposition calculus from Horn premises generates a Horn conclusion.

Exercise 13.6: We call an equational clause *happy* if it contains at least one positive literal.

(a) Prove that every inference of the superposition calculus from happy premises generates a happy conclusion.

(b) Using part (a) and the refutational completeness of superposition, prove that all sets N of happy clauses are satisfiable.

(c) Re-prove the result of part (b) using basic model theory.

Exercise 13.7 ($*$)**:** Find an unsatisfiable clause set N consisting of two unit clauses $s \approx t$ and $u \not\approx v$ and a term ordering $\succ$ such that the only nonredundant inference that does not violate the ordering restrictions of the superposition calculus is a "Positive Superposition" inference in which the left-hand side of $s \approx t$ is unified with the left-hand side of a renamed copy of $s \approx t$.

Exercise 13.8 ($*$)**:** Prove the lifting lemma (Lemma 5.4.6) for the "Equality Resolution" inference rule.