

# Automated Theorem Proving

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based on exercises by Dr. Uwe Waldmann

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## Exercises 12: Superposition

**Exercise 12.1:** Refute the following set of equational clauses by superposition:

$$x \approx b \vee x \approx c \vee x \approx d \quad (1)$$

$$e \not\approx b \quad (2)$$

$$e \not\approx c \quad (3)$$

$$e \not\approx d \quad (4)$$

Choose an appropriate ordering and perform only inferences that satisfy the ordering restrictions.

**Exercise 12.2:** Refute the following set of equational clauses by superposition:

$$f(x) \not\approx c \vee f(x) \approx b \quad (1)$$

$$f(f(x)) \approx x \quad (2)$$

$$b \not\approx c \quad (3)$$

Choose an appropriate ordering and perform only inferences that satisfy the ordering restrictions.

**Exercise 12.3:** Consider the following set of equational clauses:

$$f(b) \approx \text{true} \quad (1)$$

$$f(x) \not\approx \text{true} \vee f(g(x)) \approx \text{true} \quad (2)$$

(a) Saturate this set by computing superposition inferences *ignoring ordering restrictions*.

(b) Choose an appropriate ordering and perform only inferences that satisfy the ordering restrictions.

**Exercise 12.4:** Prove that the ground “Equality Resolution” inference rule is sound:

$$\text{Equality Resolution: } \frac{C' \vee s \not\approx s}{C'}$$

**Exercise 12.5:** Prove that the ground “Equality Factoring” inference rule is sound:

$$\text{Equality Factoring: } \frac{C' \vee s \approx t' \vee s \approx t}{C' \vee t \not\approx t' \vee s \approx t'}$$

**Exercise 12.6:** Prove that the ground “Negative Superposition” inference rule is sound:

$$\text{Neg. Superposition: } \frac{D' \vee t \approx t' \quad C' \vee s[t] \not\approx s'}{D' \vee C' \vee s[t'] \not\approx s'}$$

**Exercise 12.7:** In the lecture notes, it is stated that the ordering restrictions of the inference rules of the superposition calculus must be satisfied *after applying the mgu to the premises*. Give a simple example that shows that a literal may be maximal in a clause but that the maximality requirement may be violated after applying the mgu.

**Exercise 12.8 (\*):** Find a small unsatisfiable set  $N$  of equational clauses and a term ordering  $\succ$  such that  $N$  is saturated w.r.t. the superposition calculus *excluding* the “Equality Factoring” rule and  $N$  does not contain  $\perp$ . The existence of such a set implies that the superposition calculus is incomplete without “Equality Factoring.”

Hint: Recall the informal motivation for adding “Equality Factoring” to the calculus.