Automated Theorem Proving

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based on exercises by Dr. Uwe Waldmann

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Exercises 8: Semantic Tableaux

Exercise 8.1: Show unsatisfiability of the set of formulas

$$P \rightarrow (Q \rightarrow R) \qquad (1)$$

$$P \rightarrow Q \qquad (2)$$

$$P \wedge \neg R \qquad (3)$$

by exhibiting a strict tableau.

Exercise 8.2: Check whether the following propositional formulas are valid or not using semantic tableaux. Give a brief explanation. Use exactly the expansion rules given in the lecture.

(a) $(P \rightarrow Q) \rightarrow ((P \vee R) \rightarrow (Q \vee R))$

(b) $(P \vee Q) \rightarrow (P \wedge Q)$

Exercise 8.3: Check whether the following propositional formulas are valid or not using semantic tableaux. Give a brief explanation. Use exactly the expansion rules given in the lecture.

(a) $(P \rightarrow Q) \rightarrow ((Q \rightarrow R) \rightarrow (P \rightarrow R))$.

(b) $(R \wedge (R \rightarrow P)) \rightarrow (P \wedge \neg Q)$.

Exercise 8.4: Determine the satisfiability of the following set of ground formulas using the tableau calculus:

$$P(b) \land \neg P(d) \qquad (1)$$

$$P(c) \lor \big(P(b) \land P(d)\big) \qquad (2)$$

$$P(c) \to \neg\big(P(b) \lor P(d)\big) \qquad (3)$$

Use exactly the expansion rules given in the lecture. State explicitly whether the set is satisfiable and give an explanation for that statement.

Exercise 8.5: Extend the tableau calculus to support the following connectives:

- The Sheffer stroke, denoted $|$, is a binary connective meaning "not both." Thus, $F \,|\, G$ is equivalent to $\neg F \lor \neg G$.
- The Peirce arrow, denoted $\downarrow$, is a binary connective meaning "neither nor." Thus, $F \downarrow G$ is equivalent to $\neg F \land \neg G$.

Exercise 8.6: Refute the following set of formulas using the tableau calculus with ground instantiation:

$$\forall x\, \exists y\, P(x, y) \qquad (1)$$

$$\exists z\, \forall w\, \neg P(f(z), w) \qquad (2)$$

Exercise 8.7: Refute the following set of formulas using the free-variable tableau calculus:

$$\forall x\, \exists y\, P(x, y) \qquad (1)$$

$$\exists z\, \forall w\, \neg P(f(z), w) \qquad (2)$$