Retake Examination in the Course Automated Theorem Proving

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For convenience, a handout is provided with the definitions of the main calculi and concepts covered in the course.

Last name (in CAPITAL LETTERS):

First name (in CAPITAL LETTERS):

Matriculation number:

Program of study:

☐ **Please check with an X *only* if the exam should be voided and not graded.**

Bitte *nur* ankreuzen, wenn die Klausur entwertet und nicht korrigiert werden soll.

If your program of study does not include a voiding rule, an X mark will lead to a failing grade.

Hereby I confirm the correctness of the above information: ______________________

Signature

With your signature, you confirm that you are in sufficiently good health at the beginning of the examination and that you accept this examination bindingly.

Please leave the following table blank:

Question	1	2	3	4	5	6	$\sum$
Points	18	16	14	18	16	18	100
Score							

Instructions

You have **120 minutes** at your disposal. Written or electronic aids (except normal watches) are not permitted. Carrying electronic devices, even switched off, will be considered cheating.

Write your full name and matriculation number clearly legible on the cover sheet, as well as your name in the header on each sheet. Hand in all sheets. Leave them stapled together. Use only **pens** and **neither** the color **red nor green**.

Check that you have received all the sheets and the handout. The questions can be found on **pages 3–10**. You may use the back of the sheets for secondary calculations. If you use the back of a sheet to answer, clearly mark what belongs to which question and indicate in the corresponding question where all parts of your answer can be found. Cross out everything that should not be graded.

There are 6 questions for a total of 100 points.

Question 1 (18 points)**:** The first-order formula $F = \exists x\, (D(x) \to \forall y\, D(y))$ over $\Sigma = (\{b/0\}, \{D/1\})$ is called the *drinker formula*. It is valid. Informally: "There exists a person x such that if x drinks, then everybody drinks."

(a) Use the (unordered) resolution calculus to prove F. You will first need to transform F first to obtain a problem in clausal normal form. Spell out all your steps.

(b) Use the tableau calculus with ground instantiation to prove F. Use exactly the expansion rules seen in the lecture. Document all your steps.

Question 2 (16 points)**:** Recall that a relation $\to$ is called

terminating if there is no infinite descending chain $b_0 \to b_1 \to b_2 \to \cdots$;

normalizing if every $b \in A$ has a normal form;

locally confluent if $b \leftarrow a \to c$ implies there is a d such that $b \to^* d \leftarrow^* c$;

confluent if $b \leftarrow^* a \to^* c$ implies there is a d such that $b \to^* d \leftarrow^* c$.

Let

$$\{g(x, y, z) \to g(y, z, x),\ g(x, y, z) \to x\}$$

be a rewrite system over $\Sigma = (\{g/3,\ b/0,\ c/0,\ d/0\},\ \emptyset)$. Is the system (a) terminating? (b) normalizing? (c) locally confluent? (d) confluent? Briefly explain each of your answers.

Question 3 (14 points)**:** Apply the Knuth–Bendix procedure to the set of equations

$$g(s(x), y) \approx s(g(x, y)) \qquad (1)$$
$$g(x, s(y)) \approx s(g(x, y)) \qquad (2)$$

and transform it into a finite convergent term rewrite system. Use the Knuth–Bendix ordering with weight 1 for all symbols and variables and the precedence $g \succ s$. Document all the steps of the procedure.

Question 4 (18 points)**:** Let $\Sigma = (\{f/2,\ g/2,\ h/1,\ b/0\}, \emptyset)$. Consider the following set of equations:

$$\begin{aligned} f(b, y) &\approx y \\ f(h(x), y) &\approx h(f(x, y)) \\ g(b, y) &\approx b \\ g(h(x), y) &\approx f(y, g(x, y)) \end{aligned}$$

(a) Specify a precedence for the lpo so that the left-hand side of each equation in the set is greater than the corresponding right-hand side according to that instance of the lpo. There is no need to explain your answer.

(b) Explain why the kbo cannot be used to orient all of these equations from left to right.

(c) Specify a precedence, symbol weights, and a variable weight for the kbo so that the left-hand side of each of the *first three* equations is greater than the corresponding right-hand side according to that instance of the kbo. There is no need to explain your answer.

Question 5 (16 points)**:** For this question, we use the lpo with the precedence $k \succ j \succ i \succ f \succ d \succ c \succ b$ as the term ordering.

(a) Fill in the table below by computing the rewrite systems R_C and the increments E_C for the set of ground clauses N consisting of

$$i \approx b \vee j \approx c \qquad (1)$$
$$f(i) \not\approx f(b) \vee k \approx d \qquad (2)$$
$$i \approx b \qquad (3)$$
$$f(i) \not\approx f(b) \vee k \not\approx d \qquad (4)$$
$$i \not\approx b \vee j \approx c \qquad (5)$$

Iter.	Clause C	R_C	E_C
0	$i \approx b$	$\emptyset$	$\{i \to b\}$
1			
2			
3			
4			

(b) Which is the smallest clause $C \in N$ such that C is neither productive nor true in R_C?

(c) Is N saturated up to redundancy? Explain briefly.

Question 6 (18 points)**:** Let us call a clause *grumpy* if it contains at least one negative literal.

(a) Prove that every inference of the resolution calculus from grumpy premises generates a grumpy conclusion.

(b) Using the result stated in part (a) and the refutational completeness of resolution, prove that all sets of grumpy clauses are satisfiable. Hint: $\perp$ is not grumpy.

(c) Re-prove the result of part (b) using basic model theory. Specifically, for a given set N of grumpy clauses, exhibit an algebra $\mathcal{A}$ such that $\mathcal{A} \models N$.