First Examination in the Course Automated Theorem Proving

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For convenience, a handout is provided with the definitions of the main calculi and concepts covered in the course.

Last name (in CAPITAL LETTERS):

First name (in CAPITAL LETTERS):

Matriculation number:

Program of study:

☐ **Please check with an X *only* if the exam should be voided and not graded.**

Bitte *nur* ankreuzen, wenn die Klausur entwertet und nicht korrigiert werden soll.

If your program of study does not include a voiding rule, an X mark will lead to a failing grade.

Hereby I confirm the correctness of the above information:

Signature

With your signature, you confirm that you are in sufficiently good health at the beginning of the examination and that you accept this examination bindingly.

Please leave the following table blank:

Question	1	2	3	4	5	6	$\sum$
Points	14	24	16	16	14	16	100
Score							

Instructions

You have **120 minutes** at your disposal. Written or electronic aids (except normal watches) are not permitted. Carrying electronic devices, even switched off, will be considered cheating.

Write your full name and matriculation number clearly legible on the cover sheet, as well as your name in the header on each sheet. Hand in all sheets. Leave them stapled together. Use only **pens** and **neither** the color **red nor green**.

Check that you have received all the sheets and the handout. The questions can be found on **pages 3–10**. You may use the back of the sheets for secondary calculations. If you use the back of a sheet to answer, clearly mark what belongs to which question and indicate in the corresponding question where all parts of your answer can be found. Cross out everything that should not be graded.

There are 6 questions for a total of 100 points.

Question 1 (14 points)**:** (a) A clause is called *Horn* if it contains at most one positive literal. Prove that every inference of the resolution calculus from Horn premises generates a Horn conclusion.

(b) A clause is called *dual-Horn* if it contains at most one negative literal. Prove that every inference of the resolution calculus from dual-Horn premises generates a dual-Horn conclusion.

(c) Let N be a clause set consisting exclusively of Horn and dual-Horn clauses. Does $Res(N)$ also consist exclusively of Horn and dual-Horn clauses? Explain your answer.

Question 2 (24 points)**:** Consider the following entailment between two propositional formulas:

$$(P \wedge Q) \rightarrow R \models (P \rightarrow R) \vee (Q \rightarrow R)$$

(a) Use the DPLL procedure to prove the entailment. You will first need to express the entailment as a set of clauses using the CNF transformation. Document each CNF and DPLL step.

(b) Use the (unordered) resolution calculus to derive the empty clause from the clause set $N = \{\neg P \vee \neg Q \vee R,\, P,\, Q,\, \neg R\}$. Document all of your steps.

(c) Use the ground tableau calculus to prove the entailment

$$(P \wedge Q) \rightarrow R \models (P \rightarrow R) \vee (Q \rightarrow R)$$

You will first need to transform the entailment into a formula to refute. Use exactly the expansion rules seen in the lecture. Document all of your steps.

Question 3 (16 points)**:** Let $\Sigma = (\{f/1,\ g/2,\ h/2,\ b/0\}, \emptyset)$. Consider the following set of equations:

$$\begin{aligned} h(b, y) &\approx y \\ h(g(x, y), z) &\approx g(x, h(y, z)) \\ f(b) &\approx b \\ f(g(x, y)) &\approx h(f(y), g(x, b)) \end{aligned}$$

(a) Specify a precedence for the lpo so that the left-hand side of each equation in the set is greater than the corresponding right-hand side according to that instance of the lpo. There is no need to explain your answer.

(b) Note that the kbo cannot be used to orient the last equation from left to right. Regardless of the weights chosen, the right-hand side of the last equation has a greater weight than the left-hand side. So let us ignore this equation.

Specify a precedence, symbol weights, and a variable weight for the kbo so that the left-hand side of each of the *first three* equations is greater than the corresponding right-hand side according to that instance of the kbo. There is no need to explain your answer.

Question 4 (16 points)**:** Let $\Sigma = (\Omega, \emptyset)$ with $\Omega = \{b/0,\ c/0,\ f/2,\ g/1,\ h/2\}$ and let R be the following rewrite system:

$$\begin{aligned} h(y,c) &\to g(y) && (1)\\ f(x,x) &\to g(h(x,b)) && (2)\\ h(h(x,b),x) &\to f(x,c) && (3) \end{aligned}$$

(a) Find a kbo $\succ$ such that $\to_R \subseteq \succ$. Specify the weights and precedence of the ordering. There is no need to explain your answer.

(b) Compute all critical pairs between rules in R and determine whether they are joinable in R. There should be two critical pairs.

Question 5 (14 points)**:** Refute the following set of equational clauses using the superposition calculus:

$$f(x) \approx f(c) \lor f(x) \approx f(b) \quad (1)$$
$$f(d) \not\approx f(b) \quad (2)$$
$$f(d) \not\approx f(c) \quad (3)$$

Use the lpo with the precedence $f \succ d \succ c \succ b$. Document all of your steps.

Question 6 (16 points)**:** For this question, we use the signature $\Sigma = (\{f/1,\ b/0,\ c/0\}, \emptyset)$, the clause set $N = \{f(x) \approx b\}$ over Σ, and the kbo with $f \succ c \succ b$ and weights 1 for all symbols and variables.

(a) Complete the following table summarizing the first five iterations of the candidate interpretation construction. Hint: Recall that the clauses considered in the second column must be groundings of clauses in N.

Iter.	Clause C	R_C	E_C
0	$f(b) \approx b$		$\{f(b) \rightarrow b\}$
1			
2			
3	$f(f(c)) \approx b$		
4			

(b) Determine R_∞ for the clause set N.