

Mockup Examination in the Course Automated Theorem Proving

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For convenience, a handout is provided with the definitions of the main calculi and concepts covered in the course.

Last name (in CAPITAL LETTERS):

First name (in CAPITAL LETTERS):

Matriculation number:

Program of study:

- ☐ Please check with an X *only* if the exam should be voided and not graded.

Bitte *nur* ankreuzen, wenn die Klausur entwertet und nicht korrigiert werden soll.

If your program of study does not include a voiding rule, an X mark will lead to a failing grade.

Hereby I confirm the correctness of the above information:

Signature

With your signature, you confirm that you are in sufficiently good health at the beginning of the examination and that you accept this examination bindingly.

Please leave the following table blank:

Question	1	2	3	4	5	6	Σ
Points	16	20	10	20	16	18	100
Score							

Instructions

You have **120 minutes** at your disposal. Written or electronic aids (except normal watches) are not permitted. Carrying electronic devices, even switched off, will be considered cheating.

Write your full name and matriculation number clearly legible on the cover sheet, as well as your name in the header on each sheet. Hand in all sheets. Leave them stapled together. Use only **pens** and **neither** the color **red nor green**.

Check that you have received all the sheets and the handout. The questions can be found on **pages 3–8**. You may use the back of the sheets for secondary calculations. If you use the back of a sheet to answer, clearly mark what belongs to which question and indicate in the corresponding question where all parts of your answer can be found. Cross out everything that should not be graded.

There are 6 questions for a total of 100 points.

Question 1 (16 points): Let

$$F = \exists z \forall x ((\exists y P(z, y) \rightarrow \exists y Q(x, y)))$$

be a first-order formula.

(a) Transform F into clausal normal form. Proceed one step at a time and record each step.

(b) Are the formula F and its clausal normal form equivalent? Explain your answer.

Question 2 (20 points): Let N be the following set of propositional clauses over $\Pi = \{P, Q, R\}$:

$$P \vee \neg Q \quad (1)$$

$$P \vee \neg R \quad (2)$$

$$\neg P \vee R \quad (3)$$

(a) Use the DPLL procedure to find a (total) model of N . Document each step of the procedure.

(b) Use the DPLL procedure to prove that $N \models Q \rightarrow R$. Document each step of the procedure.

Question 3 (10 points): Let $\Sigma = (\Omega, \emptyset)$ with $\Omega = \{f/1, g/1, h/2, b/0, c/0\}$. Find a total precedence \succ on Ω such that the lexicographic path ordering \succ_{lpo} that is induced by \succ satisfies the following three properties simultaneously.

$$h(x, f(y)) \succ_{\text{lpo}} h(g(y), y) \quad (1)$$

$$h(x, c) \succ_{\text{lpo}} g(h(x, b)) \quad (2)$$

$$g(b) \succ_{\text{lpo}} c \quad (3)$$

There is no need to explain your answer.

Question 4 (20 points): Let $\Sigma = (\{f/1, g/2, b/0, c/0\}, \{P/2, Q/1, R/2\})$. Let the atom ordering \succ be the kbo with weight 1 for all symbols from Σ and variables and the precedence $P \succ Q \succ R \succ f \succ g \succ b \succ c$. Let N be the following set of clauses over Σ :

$$\neg R(b, x) \quad (1)$$

$$Q(g(x, z)) \vee Q(g(b, f(y))) \vee \neg R(x, z) \quad (2)$$

$$P(f(x), x) \vee \neg Q(f(c)) \vee \neg R(x, c) \quad (3)$$

$$\neg P(x, c) \vee Q(x) \quad (4)$$

$$\neg P(x, b) \vee R(c, y) \quad (5)$$

(a) Suppose that the selection function sel selects no literals. Compute all Res_{sel}^\prec inferences between the clauses (1)–(5). Do not compute inferences between derived clauses.

(b) Is the set N saturated up to redundancy? Explain your answer.

Question 5 (16 points): Let R be the following set of rewrite rules over $\Sigma = (\{f/1, g/2, h/1, c/0\}, \emptyset)$:

$$f(f(x)) \rightarrow h(h(x)) \quad (1)$$

$$g(f(y), x) \rightarrow g(y, x) \quad (2)$$

$$h(g(z, f(c))) \rightarrow f(z) \quad (3)$$

There are three critical pairs between the three rules. Find them.

Question 6 (18 points): Let $\Sigma = (\{f/1, g/1, b/0\}, \emptyset)$. Consider the set N consisting of the following of equational clauses over Σ :

$$\begin{aligned} g(b) &\approx b \\ g(x) &\not\approx b \vee g(f(x)) \approx b \end{aligned}$$

We use the superposition calculus without selection and the kbo with $g \succ f \succ b$ and weight 1 for symbols and variables as the term ordering.

(a) Is the set N saturated up to redundancy? If yes, briefly explain why. If no, give an inference from N that is not redundant w.r.t. N .

(b) Complete the following table documenting the first four iterations of the candidate interpretation construction leading to the rewrite system R_∞ for the set of ground clauses $G_\Sigma(N)$.

Iter.	Clause C	R_C	E_C
0	$g(b) \approx b$	\emptyset	
1	$g(b) \not\approx b \vee g(f(b)) \approx b$		
2	$g(f(b)) \not\approx b \vee g(f(f(b))) \approx b$		
3	$g(g(b)) \not\approx b \vee g(f(g(b))) \approx b$		

