Aufgabe 12-3. Eine mögliche Lösung ist:

$$
\text{(Gen)}\;\dfrac{\text{(Fn)}\;\dfrac{\text{(Fn)}\;\dfrac{\text{(Fn)}\;\dfrac{\text{(Sub)}\;\dfrac{\text{(App)}\;\dfrac{\text{(Var)}\;\dfrac{}{[\mathtt{f} \mapsto (\alpha \xrightarrow{\varepsilon_1} \beta), \mathtt{g} \mapsto (\beta \xrightarrow{\varepsilon_2} \gamma), \mathtt{x} \mapsto \alpha] \vdash \mathtt{g}\colon (\beta \xrightarrow{\varepsilon_2} \gamma) \mathbin{\&} \varnothing} \qquad \Pi}{[\mathtt{f} \mapsto (\alpha \xrightarrow{\varepsilon_1} \beta), \mathtt{g} \mapsto (\beta \xrightarrow{\varepsilon_2} \gamma), \mathtt{x} \mapsto \alpha] \vdash \mathtt{g}\ (\mathtt{f}\ \mathtt{x})\colon \gamma \mathbin{\&} \varepsilon_2 \cup \varnothing \cup \varepsilon_1}}{[\mathtt{f} \mapsto (\alpha \xrightarrow{\varepsilon_1} \beta), \mathtt{g} \mapsto (\beta \xrightarrow{\varepsilon_2} \gamma), \mathtt{x} \mapsto \alpha] \vdash \mathtt{g}\ (\mathtt{f}\ \mathtt{x})\colon \gamma \mathbin{\&} \varepsilon_1 \cup \varepsilon_2}}{[\mathtt{f} \mapsto (\alpha \xrightarrow{\varepsilon_1} \beta), \mathtt{g} \mapsto (\beta \xrightarrow{\varepsilon_2} \gamma)] \vdash \mathtt{fn\ x}\ \Rightarrow \mathtt{g}\ (\mathtt{f}\ \mathtt{x})\colon \alpha \xrightarrow{\varepsilon_1 \cup \varepsilon_2} \gamma \mathbin{\&} \varnothing}}{[\mathtt{f} \mapsto (\alpha \xrightarrow{\varepsilon_1} \beta)] \vdash \mathtt{fn\ g}\ \Rightarrow \mathtt{fn\ x}\ \Rightarrow \mathtt{g}\ (\mathtt{f}\ \mathtt{x})\colon (\beta \xrightarrow{\varepsilon_2} \gamma) \xrightarrow{\varnothing} (\alpha \xrightarrow{\varepsilon_1 \cup \varepsilon_2} \gamma) \mathbin{\&} \varnothing}}{\emptyset \vdash \mathtt{fn\ f}\ \Rightarrow \mathtt{fn\ g}\ \Rightarrow \mathtt{fn\ x}\ \Rightarrow \mathtt{g}\ (\mathtt{f}\ \mathtt{x})\colon (\alpha \xrightarrow{\varepsilon_1} \beta) \xrightarrow{\varnothing} (\beta \xrightarrow{\varepsilon_2} \gamma) \xrightarrow{\varnothing} (\alpha \xrightarrow{\varepsilon_1 \cup \varepsilon_2} \gamma) \mathbin{\&} \varnothing}}{\emptyset \vdash \mathtt{fn\ f}\ \Rightarrow \mathtt{fn\ g}\ \Rightarrow \mathtt{fn\ x}\ \Rightarrow \mathtt{g}\ (\mathtt{f}\ \mathtt{x})\colon \forall \alpha, \beta, \gamma, \varepsilon_1, \varepsilon_2.\, (\alpha \xrightarrow{\varepsilon_1} \beta) \xrightarrow{\varnothing} (\beta \xrightarrow{\varepsilon_2} \gamma) \xrightarrow{\varnothing} (\alpha \xrightarrow{\varepsilon_1 \cup \varepsilon_2} \gamma) \mathbin{\&} \varnothing}
$$

Darin steht Π für folgende Herleitung:

$$
\text{(Sub)}\;\dfrac{\text{(App)}\;\dfrac{\text{(Var)}\;\dfrac{}{[\mathtt{f} \mapsto (\alpha \xrightarrow{\varepsilon_1} \beta), \mathtt{g} \mapsto (\beta \xrightarrow{\varepsilon_2} \gamma), \mathtt{x} \mapsto \alpha] \vdash \mathtt{f}\colon (\alpha \xrightarrow{\varepsilon_1} \beta) \mathbin{\&} \varnothing} \qquad \text{(Var)}\;\dfrac{}{[\mathtt{f} \mapsto (\alpha \xrightarrow{\varepsilon_1} \beta), \mathtt{g} \mapsto (\beta \xrightarrow{\varepsilon_2} \gamma), \mathtt{x} \mapsto \alpha] \vdash \mathtt{x}\colon \alpha \mathbin{\&} \varnothing}}{[\mathtt{f} \mapsto (\alpha \xrightarrow{\varepsilon_1} \beta), \mathtt{g} \mapsto (\beta \xrightarrow{\varepsilon_2} \gamma), \mathtt{x} \mapsto \alpha] \vdash \mathtt{f\ x}\colon \beta \mathbin{\&} \varepsilon_1 \cup \varnothing \cup \varnothing}}{[\mathtt{f} \mapsto (\alpha \xrightarrow{\varepsilon_1} \beta), \mathtt{g} \mapsto (\beta \xrightarrow{\varepsilon_2} \gamma), \mathtt{x} \mapsto \alpha] \vdash \mathtt{f\ x}\colon \beta \mathbin{\&} \varepsilon_1}
$$

Wenn man $\varnothing$ vermeiden will, können die Vorkommen von $\varnothing$ in den Typannotaten mithilfe der Regel (Sub) auch durch Effektvariablen ersetzt werden.