

### Aufgabe 9-2.

Abkürzungen:

$\Gamma$  steht für  $[x \mapsto \text{int}, i \mapsto \forall \alpha. \text{int} \rightarrow (\alpha \rightarrow \alpha) \rightarrow (\alpha \rightarrow \alpha)]$ .

$I(\alpha)$  steht für  $(\alpha \rightarrow \alpha) \rightarrow (\alpha \rightarrow \alpha)$ .

$$\begin{array}{c}
 \text{(Gen)} \frac{\text{Annahme}}{[x \mapsto \text{int}] \vdash e: \text{int} \rightarrow (\alpha \rightarrow \alpha) \rightarrow (\alpha \rightarrow \alpha)} \quad \text{(App)} \frac{\Pi \quad \text{(Const)} \frac{}{\Gamma \vdash 0: \text{int}}}{\Gamma \vdash i x (i x) (\text{fn } x \Rightarrow x + 1) 0: \text{int}} \\
 \text{(Let)} \frac{[x \mapsto \text{int}] \vdash e: \forall \alpha. \text{int} \rightarrow (\alpha \rightarrow \alpha) \rightarrow (\alpha \rightarrow \alpha)}{[x \mapsto \text{int}] \vdash \text{let } i = e \text{ in } i x (i x) (\text{fn } x \Rightarrow x + 1) 0: \text{int}} \\
 \text{(Fn)} \frac{[x \mapsto \text{int}] \vdash \text{let } i = e \text{ in } i x (i x) (\text{fn } x \Rightarrow x + 1) 0: \text{int}}{\emptyset \vdash \text{fn } x \Rightarrow \text{let } i = e \text{ in } i x (i x) (\text{fn } x \Rightarrow x + 1) 0: \text{int} \rightarrow \text{int}}
 \end{array}$$

Die Herleitung  $\Pi$  ist:

$$\begin{array}{c}
 \text{(Var)} \frac{\Gamma(i) = \forall \alpha. \text{int} \rightarrow I(\alpha)}{\Gamma \vdash i: \forall \alpha. \text{int} \rightarrow I(\alpha)} \quad \text{(Var)} \frac{\Gamma(x) = \text{int}}{\Gamma \vdash x: \text{int}} \quad \text{(Var)} \frac{\Gamma(i) = \forall \alpha. \text{int} \rightarrow I(\alpha)}{\Gamma \vdash i: \forall \alpha. \text{int} \rightarrow I(\alpha)} \quad \text{(Var)} \frac{\Gamma(x) = \text{int}}{\Gamma \vdash x: \text{int}} \\
 \text{(Inst)} \frac{\Gamma \vdash i: \forall \alpha. \text{int} \rightarrow I(\alpha)}{\Gamma \vdash i: \text{int} \rightarrow I(\text{int}) \rightarrow I(\text{int})} \quad \text{(Inst)} \frac{\Gamma \vdash i: \forall \alpha. \text{int} \rightarrow I(\alpha)}{\Gamma \vdash i: \text{int} \rightarrow I(\text{int})} \quad \text{(Inst)} \frac{\Gamma \vdash i: \forall \alpha. \text{int} \rightarrow I(\alpha)}{\Gamma \vdash i: \text{int} \rightarrow I(\text{int})} \quad \text{(Inst)} \frac{\Gamma \vdash i: \forall \alpha. \text{int} \rightarrow I(\alpha)}{\Gamma \vdash i: \text{int} \rightarrow I(\text{int})} \\
 \text{(App)} \frac{\Gamma \vdash i: \text{int} \rightarrow I(\text{int}) \rightarrow I(\text{int}) \quad \Gamma \vdash x: \text{int}}{\Gamma \vdash i x (i x): (\text{int} \rightarrow \text{int}) \rightarrow \text{int} \rightarrow \text{int}} \quad \text{(App)} \frac{\Gamma \vdash i: \text{int} \rightarrow I(\text{int}) \quad \Gamma \vdash x: \text{int}}{\Gamma \vdash i x (i x): (\text{int} \rightarrow \text{int}) \rightarrow \text{int} \rightarrow \text{int}} \\
 \text{(App)} \frac{\Gamma \vdash i x (i x): (\text{int} \rightarrow \text{int}) \rightarrow \text{int} \rightarrow \text{int} \quad \Gamma \vdash x + 1: \text{int}}{\Gamma \vdash i x (i x) (\text{fn } x \Rightarrow x + 1) 0: \text{int} \rightarrow \text{int}} \quad \text{(App)} \frac{\Gamma \vdash x: \text{int} \quad \Gamma \vdash 1: \text{int}}{\Gamma \vdash x + 1: \text{int}} \\
 \text{(App)} \frac{\Gamma \vdash i x (i x) (\text{fn } x \Rightarrow x + 1) 0: \text{int} \rightarrow \text{int} \quad \Gamma \vdash x + 1: \text{int}}{\Gamma \vdash \text{fn } x \Rightarrow x + 1: \text{int} \rightarrow \text{int}}
 \end{array}$$

Beachte:

$$\begin{aligned}
 I(\text{int}) \rightarrow I(\text{int}) &= ((\text{int} \rightarrow \text{int}) \rightarrow (\text{int} \rightarrow \text{int})) \rightarrow ((\text{int} \rightarrow \text{int}) \rightarrow (\text{int} \rightarrow \text{int})) \\
 &= I(\text{int} \rightarrow \text{int})
 \end{aligned}$$