Aufgabe 8-1.
Abkürzungen: τ steht für $\texttt{int} \to \texttt{int}$ und Γ steht für $[f \mapsto (\tau \to \tau) \to \tau, x \mapsto (\tau \to \tau)]$.

$$\dfrac{\dfrac{\dfrac{\Gamma(x) = \tau \to \tau}{\Gamma \vdash x\colon\ \tau \to \tau}\,(\text{Var}) \quad \dfrac{\dfrac{\Gamma(f) = (\tau \to \tau) \to \tau}{\Gamma \vdash f\colon\ (\tau \to \tau) \to \tau}\,(\text{Var}) \quad \dfrac{\Gamma(x) = \tau \to \tau}{\Gamma \vdash x\colon\ \tau \to \tau}\,(\text{Var})}{\Gamma \vdash f\ x\colon\ \tau}\,(\text{App})}{\Gamma \vdash x\ (f\ x)\ :\ \tau}\,(\text{App})}{\vdash (\texttt{fun}\ f\ x \Rightarrow x\ (f\ x))\ :\ (\tau \to \tau) \to \tau}\,(\text{Fun})$$

Abkürzung: Γ steht für $[f \mapsto (\texttt{int} \to \texttt{int}), x \mapsto \texttt{int}]$.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\Gamma(x) = \texttt{int}}{\Gamma \vdash x\colon \texttt{int}}\,(\text{Var}) \quad \dfrac{}{\Gamma \vdash 0\colon \texttt{int}}\,(\text{Const})}{\Gamma \vdash x = 0\colon \texttt{bool}}\,(\text{Op}) \quad \dfrac{}{\Gamma \vdash 1\colon \texttt{int}}\,(\text{Const}) \quad \dfrac{\dfrac{\Gamma(f) = \texttt{int} \to \texttt{int}}{\Gamma \vdash f\colon \texttt{int} \to \texttt{int}}\,(\text{Var}) \quad \dfrac{\dfrac{\dfrac{\Gamma(f) = \texttt{int} \to \texttt{int}}{\Gamma \vdash f\colon \texttt{int} \to \texttt{int}}\,(\text{Var}) \quad \dfrac{\dfrac{\Gamma(x) = \texttt{int}}{\Gamma \vdash x\colon \texttt{int}}\,(\text{Var}) \quad \dfrac{}{\Gamma \vdash 1\colon \texttt{int}}\,(\text{Const})}{\Gamma \vdash x - 1\colon \texttt{int}}\,(\text{Op})}{\Gamma \vdash f\ (x-1)\colon \texttt{int}}\,(\text{App})}{\Gamma \vdash x * (f\ (x-1))\colon \texttt{int}}\,(\text{Op})}{\Gamma \vdash \texttt{if}\ x = 0\ \texttt{then}\ 1\ \texttt{else}\ x * (f\ (x-1))\colon\ \texttt{int}}\,(\text{If})}{[f \mapsto (\texttt{int} \to \texttt{int})] \vdash \texttt{fn}\ x \Rightarrow \texttt{if}\ x = 0\ \texttt{then}\ 1\ \texttt{else}\ x * (f\ (x-1))\colon\ \texttt{int} \to \texttt{int}}\,(\text{Fn})}{\vdash \texttt{fn}\ f \Rightarrow \texttt{fn}\ x \Rightarrow \texttt{if}\ x = 0\ \texttt{then}\ 1\ \texttt{else}\ x * (f\ (x-1))\colon\ (\texttt{int} \to \texttt{int}) \to (\texttt{int} \to \texttt{int})}\,(\text{Fn})$$