Exercises on
SAT Solving
Problem set 5

Exercise 8: Construct the formulas $\langle\langle M, \varphi \rangle \rangle^k$ for $k = 2, 3$, where $M = (S, I, T, l)$ is a transition system for a 2-bit binary counter:

- $S = \{a, b, c, d\}$
- $I = \{a\}$
- $T = \{(a, b), (b, c), (c, d), (d, a)\}$
- $l(a) = \emptyset$
- $l(b) = \{\text{low}\}$
- $l(c) = \{\text{high}\}$
- $l(d) = \{\text{low}, \text{high}\}$

and $\varphi$ is the LTL formula $F(\neg \text{low} \land X \text{high})$.

Practical Exercise 7: The two Christmas elves Vee and Wedge like to play Conway’s “Game of Life” on a $7 \times 7$-torus. To their enjoyment, Santa came up with the following pattern. After three generations, a beautiful Christmas tree emerges:

However, this year there is actually snow on Christmas day, so please help Santa find the pattern that will result in a snowy Christmas tree after three generations:
Remember the rules of Game of Life:

- White cells are alive, grey cells are dead.
- Every cell has eight neighbors (wrapping around the edges = a torus).
- A live cell with fewer than two live neighbors dies.
- A live cell with two or three live neighbors lives on.
- A live cell with more than three live neighbors dies.
- A dead cell with exactly three live neighbours becomes alive.

Use a SAT-solver for your help.

**Hint:** Think of a good cell-to-variable-mapping first, then carefully encode the rules.

**Hint:** You can use the open-source program Golly\(^1\) to play with cellular automata. Above rules correspond to the setting *Control* → *Set Rule*... *B3/S23:T7,7*.

Hand in on UniWorX until Tuesday, December 22, 2015, 4pm.

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\(^1\)http://golly.sourceforge.net/